Plan: (1) Intro/Ising magnets (2) Heisenberg systems (3) Quantum Spin Liquids

~~Mott~~ 1

Frustrated Magnets

- Magnets → I will focus on nearly localized e^-'s
  $\approx$ atomic eigenstates w/ weak overlap

  usually Mott Insulators

  - Get magnetism when partially filled shells

    e.g. Mn^{2+}  $S = 5/2$

    lots of complexity + variation due to many different atomic conditions

- How do you know?
  - Usually good insulator $\ln\rho \sim 1/T^{\alpha}$
  - Curie Law at high-Temp

    $$\chi \sim A/T \qquad A = \frac{N g^2 \mu_B^2\, S(S+1)}{3 k_B} \quad \text{Curie Constant}$$

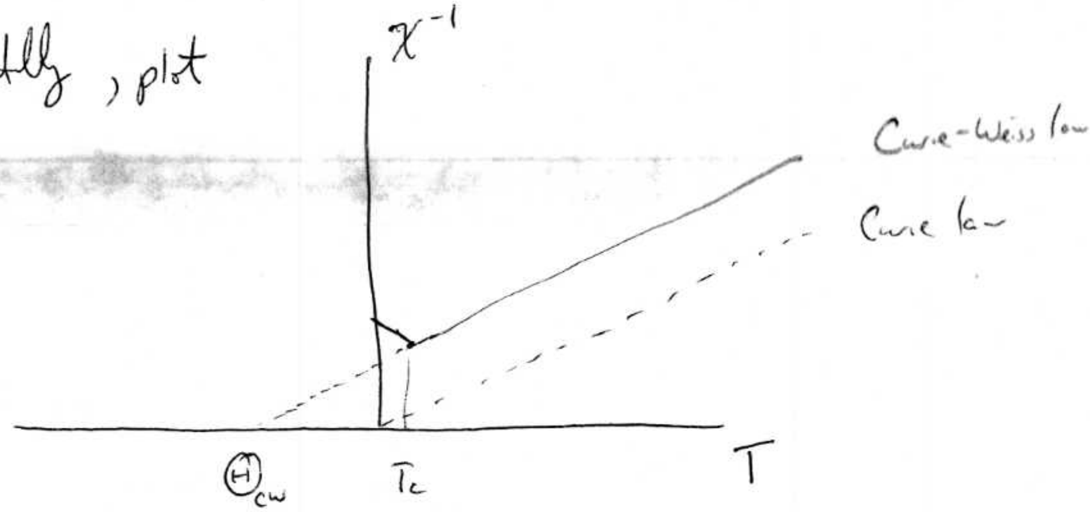
    signature of free moments.

- Interactions between spins e.g. $H = \frac{1}{2} \sum_{ij} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$

  (note I choose $J > 0$ for AF)

  - AF interactions → generally suppresses χ.

Expmtlly, plot



$$\chi \sim \frac{A}{T - \Theta_{CW}} \qquad \Theta_{CW} < 0 \quad \text{for AFs}$$

- This form is obtained in MFT above T_c
- Also in high-T expansion

$$\chi^{-1} \sim A^{-1}\left(T - \Theta_{CW} - O(1/T)\right)$$

$$\Theta_{CW} = \frac{-\left(\sum_j J_{ij}\right) S(S+1)}{3 k_B}$$

- $|\Theta_{CW}|$ Gives a measure of strength of magnetic interactions

  - For ~~unfrustrated~~ e.g. ~~sq~~ cubic lattice AF,

$$|\Theta_{CW}| = T_c^{MF}.$$

- "Frustration Fingerprint"

$$f = \frac{|\Theta_{CW}|}{T_c} \gg 1 \longrightarrow \text{ordering is anomalously suppressed. "frustrated"}$$

empirically $f \gtrsim 5\text{-}10$ usually accepted as frustrated.

Why $f \gg 1$?

- Frustration $\cong$ competing interactions

  → Cannot satisfy all $J_{ij} \vec{S}_i \cdot \vec{S}_j$ terms simultaneously

  → Get many almost equally good compromises

  = "degeneracy"

  → System fluctuates between these degenerate states instead of ordering ← Quantum / Thermal ?

Before getting to any details, talk about why this is interesting

- Emergent low energy scale $T_c \ll \Theta_{cw}$

  - Expect some new (universal?) behavior $T_c < T < \Theta_{cw}$?
  - Here spins are correlated but not ordered. "spin liquid"

- Quasi-degeneracy

  - Perturbations that break degeneracy are only intrinsic energy scale

    → Similar to FQHE → project V_c into LLL

  - Very sensitive
    - relatively small effects can control G.S.
    - controllable ?

• Interesting states arise
- non-collinear magnetic order (c.f. Mostovoy)
- non-magnetic states: dimers, spin-liquids
- reduced dimensionality

Ising Magnets

$\underline{\text{theory}}$ $\vec{S}_i \longrightarrow \sigma_i^z \equiv \sigma_i \quad \sigma_i = \pm 1 \quad ($

practice: need strong ~~single-ion anisotropy~~ single-ion anisotropy (crystal field + spin-orbit) effects

most $\overset{\text{Ising-like}}{\cancel{\text{common}}} \rightarrow$ f e^-'s in rare earth elements.

but also some d e^-s in asymmetric environments

~~Difficulty: one of Ising-like~~

Difficulties

• Ising axes often not collinear

$\vec{S}_i \longrightarrow \hat{n}_i \sigma_i \qquad \hat{n}_i$ different for different sites

• For f e^-'s, dipolar interactions important.

(both effects are present in famous "spin ice" materials I will discuss)

NN Ising AFs

$$H = J \sum_{\langle ij \rangle} \sigma_i \sigma_j$$

Note: this is purely classical problem since $[\sigma_i^z, \sigma_j^z] = 0$.

QMs arises if one includes transverse components, e.g.

$$J_\perp \left(S_i^x S_j^x + S_i^y S_j^y \right)$$

or $h_\perp S_i^x$ terms.

Let's ignore for now.

On bipartite lattice, H has unique G.S. up to symmetry

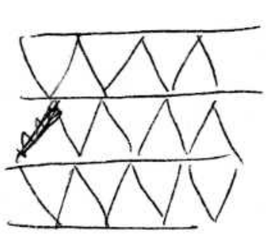
$$\begin{matrix} + & - \\ - & + \end{matrix}$$

If odd-site loops are present, get frustration

 ? $\rightarrow$ $\begin{matrix} & - & \\ + & & + \end{matrix}$ $\quad$ $\begin{matrix} & + & \\ - & & - \end{matrix}$ + perms

$\rightarrow$ 6 G.S. of triangle

Triangular (hexagonal) lattice



one ~~frustrated~~ dissatisfied bond per triangle.

sometimes call the frustrated bonds.