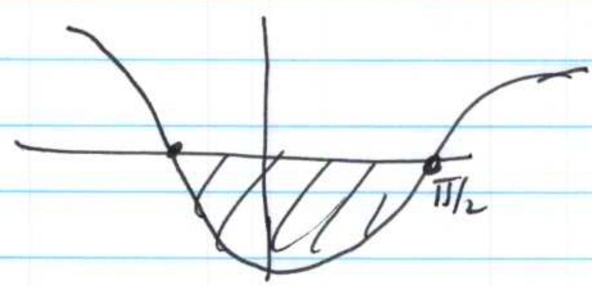
Lecture II Shankar July 2, 08

A trial run with RG.

Spinless fermions at half-filling in $d=1$

$$H_0 = -\sum \psi_n^\dagger \psi_{n+1} + hc$$

$$= -\int dK \; \psi^\dagger(K)\,\psi(K)\cos K$$



is a gapless system. Now add

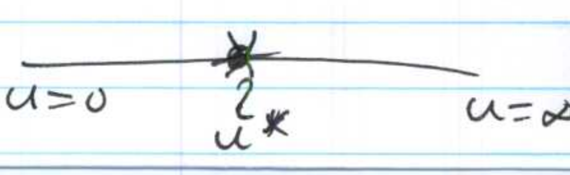
$$H_1 = U \sum \psi_n^\dagger \psi_n \,\psi_{n+1}^\dagger \psi_{n+1}$$

$U = 0$ conductor

$U \to \infty$ CDW ● ○ ● ○ ● ○

or ○ ● ○ ● ○ ●

When does it change



Mean field theory:

$\chi_{CDW} = \infty$ CDW forms at $U = 0^+$

$\chi_{BCS} = \infty$ SC " " $U = 0^-$

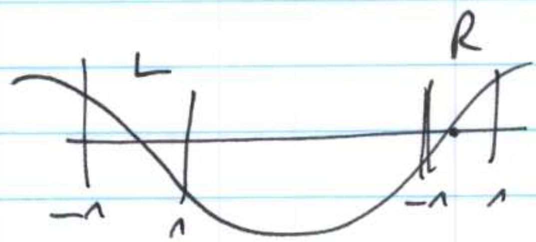
What actually happens? We use RG.

<u>Tactic</u> $H_0 \to \int d\bar{\Psi}\, d\Psi\, e^{S_0}$

Find RG such that $\{ S_0 \to S_0$ (a fixed point) $\}$

now add <u>any</u> $H_1 \to S_1$ and see if S_1 is rel, irrel. or marginal

$$H_0 = \int dk\, \psi^\dagger(k)\psi(k) \cos k$$



Keep just a band $k_F \pm \Lambda$ at L & R Fermi points. Linearize that

$$\boxed{H_0 = \sum_{\alpha = L,R} \psi^\dagger_\alpha(k)\psi_\alpha(k)\, k}$$

Recall that if

$$H = \omega_0 \psi^\dagger \psi$$

$$Z = \int [d\bar{\Psi} d\Psi]\, e^{\int \bar{\Psi}(\omega)[i\omega - \omega_0]\Psi(\omega) \frac{d\omega}{2\pi}}$$

now we have an osc. at each k and α with $\omega_0 = k$. Thus

$$Z = \int (d\bar{\Psi} d\Psi)\, e^{S_0}$$

$$S_0 = \sum_\alpha \int_{-\infty}^{\infty}\int_{-\Lambda}^{\Lambda} \bar{\Psi}(\omega k)[i\omega - k]\Psi(\omega k) \frac{d\omega\, dk}{(2\pi)^2}$$

is a Gaussian action.

RG $\Lambda \to \Lambda/s$ → change cut-off

$$\left.\begin{aligned} k' &= sk \\ \omega' &= s\omega \end{aligned}\right) \text{ rescale momenta, freq}$$

$$S_0 \to \sum_\alpha \int_{-\infty}^{\infty}\int_{-\Lambda}^{\Lambda} \bar{\Psi}\left(\tfrac{\omega'}{s}, \tfrac{k'}{s}\right)\left[\frac{i\omega' - k'}{s}\right]\Psi\left(\tfrac{\omega'}{s}, \tfrac{k'}{s}\right)\frac{d\omega'\, dk'}{s^2}$$

So we let $\underbrace{\Psi\left(\tfrac{\omega'}{s}, \tfrac{k'}{s}\right) = \Psi'(\omega', k')\, s^{3/2}}$

Then $S_0 \to S_0$ Rescale field

Why we must ensure $S_0 \to S_0$: apples → apples.