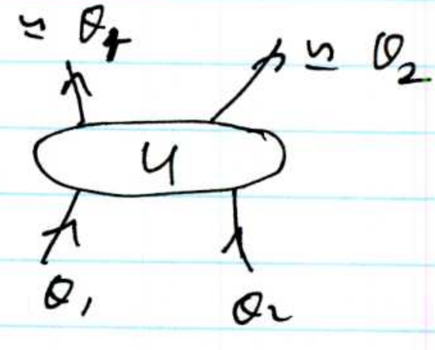
RG for fermions III Shankar 7/7/08.

Recall: For circular FS, at tree level we have just



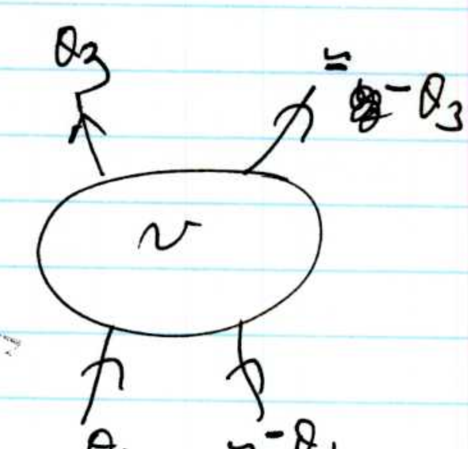
I $u(\theta_1, \theta_2) = u(\theta_1 - \theta_2) = F(\theta)$

$\theta_3 \simeq \theta_1$ i.e $\theta_3 = \theta_1 \pm \Lambda/K_F$

$\theta_4 = \theta_2 \pm \Lambda/K_F$

$u(\theta) = \sum u_m e^{im\theta}$ ↓ Landau.

i.e long as $\Lambda \neq 0$; non forward scattering is allowed, but u does not vary in this tiny range



II $v(\theta_1, \theta_3) = v(\theta_3 - \theta_1)$

BCS channel

again $\theta_2 \simeq -\theta_1 \pm \Lambda/K_F$

so BCS is not $P=0$ but $P \simeq \Lambda/K_F$.

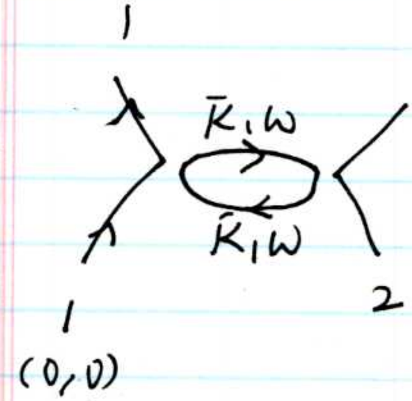
But v is evaluated at $\theta_2 = -\theta_1$; $\theta_4 = -\theta_3$.

Small wiggle room of order Λ/K_F; but v is assumed constant over that range.

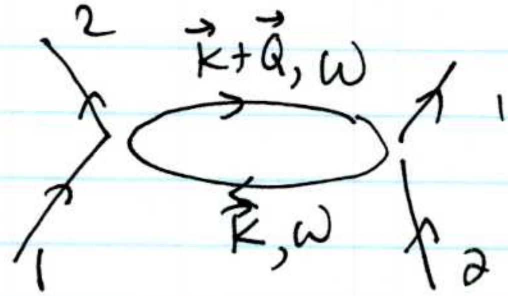
Need to go to 1 loop for u and v.

For u choose $\omega = k = 0$ at each leg.

u:

$\delta u =$ 

$= 0$ because poles on same ~~state~~ half plane, we need particle-hole

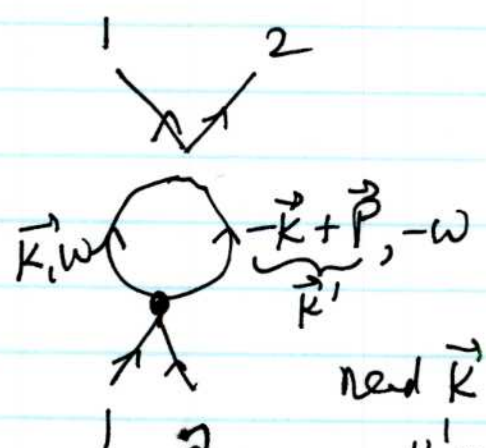
$-$ 

$Q \sim k_F$

K lies in $d\Lambda$ of $-\Lambda$, needs to absorb Q and scatter to $d\Lambda$ of $+\Lambda$.

$\delta\theta = \frac{d\Lambda}{k_F}$, $dK = d\Lambda$

$du \sim \frac{d\Lambda^2}{\Lambda}$

$-\frac{1}{2}$ 

$P \sim k_F$

need $\vec{K} \simeq 1$

$K' \simeq 2$

$\delta\theta$ is small

dK is small

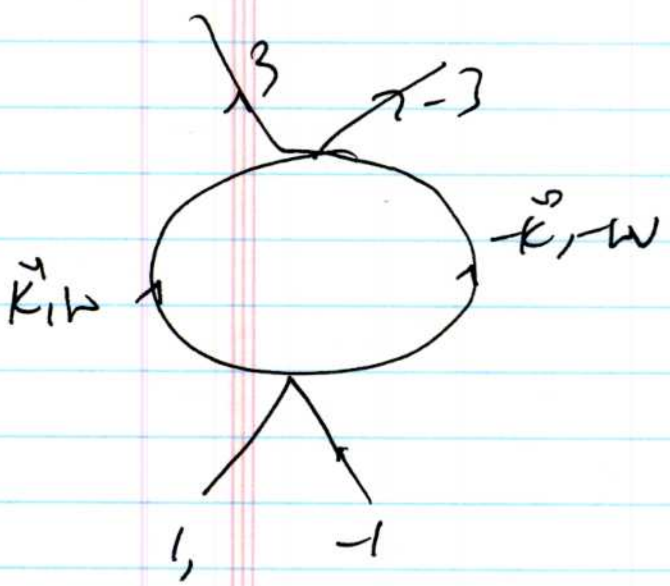
$du \sim \frac{(d\Lambda)^2}{\Lambda}$

$-\vec{K} + \vec{P} = FS$

2 choices for $-K$, $\delta\theta \sim d\Lambda/k_F$

BCS coupling

$\delta V = \delta$  $= \underbrace{ZS + ZS'}_{\frac{\delta\Lambda^2}{\Lambda}} + \underbrace{BCS}_{\text{consider}}$



if $\vec{k}$ lies in shell so does $-\vec{k}$ for all θ

$$dV(\theta_3-\theta_1) = \int d\theta \int d\omega \int_{\Lambda\pm\delta\Lambda} dk \, \frac{v(\theta-\theta_1)\, v(\theta_3-\theta)\, d\theta \, d\omega \, dk}{(i\omega - k)(-i\omega - k)}$$

$$\frac{dV}{dt} \simeq -c\int v(\theta-\theta_1)\, v(\theta_3-\theta)\, d\theta$$